DESIGN OF MACHINE ELEMENTS

Module-V

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Springs

Spring is an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed

Types

Induces torsional shear stresses in spring wire even the spring is subjected to axial loading

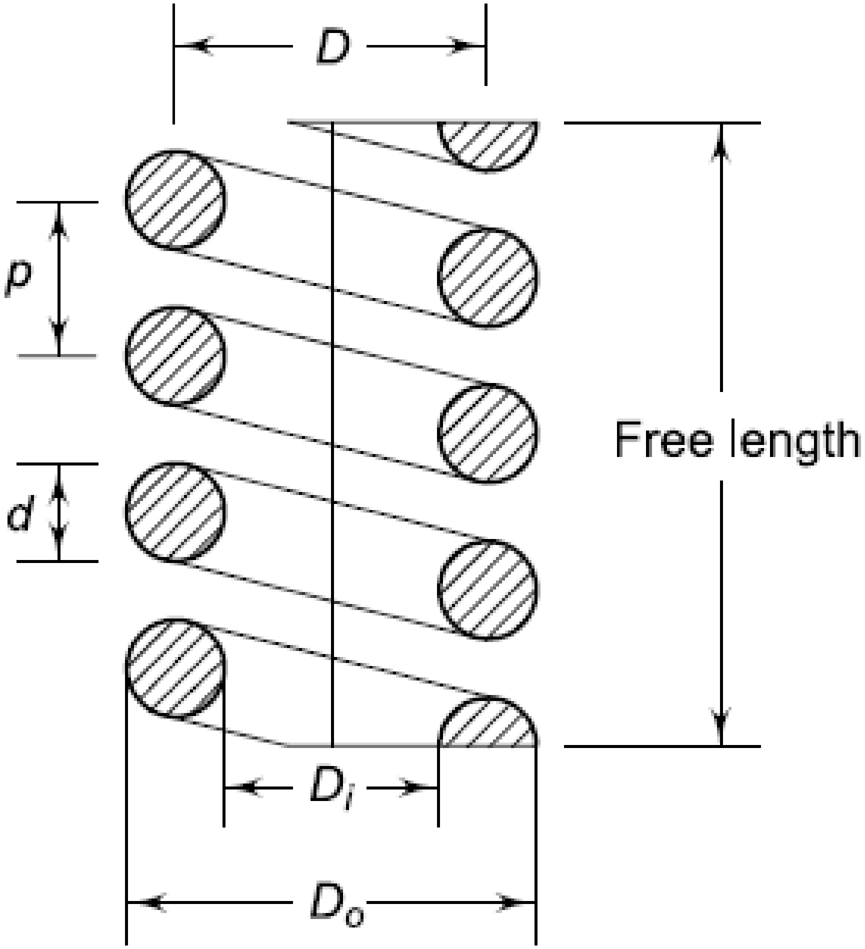
Compression Spring

Extension Spring

Leaf Spring

Torsion Spring

Terminology of Springs



Mean diameter

$$D = \frac{D_i + D_o}{2}$$

Spring index parameter

$$C = \frac{D}{d}$$ Eqn 11.2c

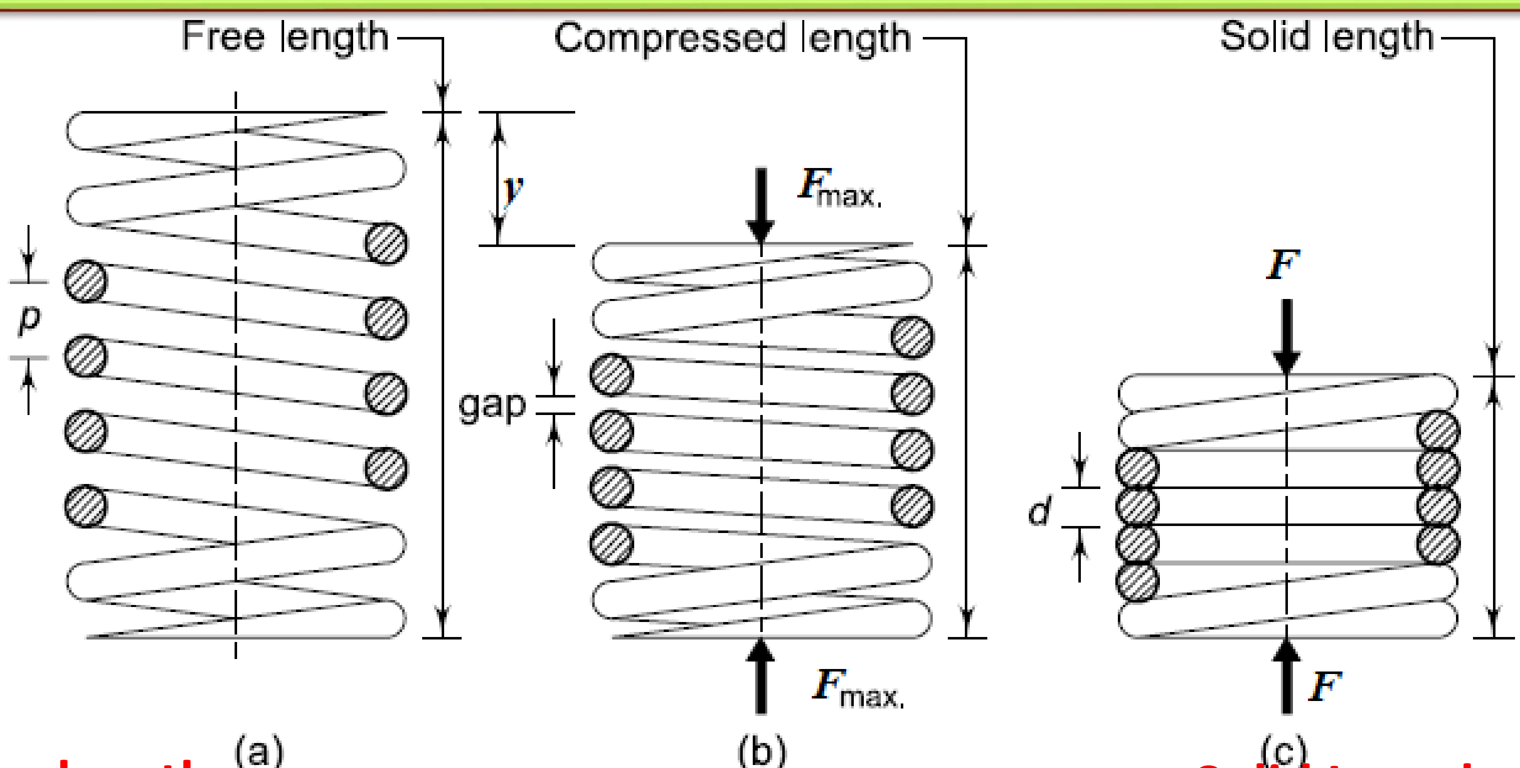
Spring index indicates the relative sharpness of curvature

If C low value: Stresses excessive due to curvature effect

If C high: Prone to buckling

C: 6-9 is preferred

Terminology of Springs



Free length:

The axial length of an unloaded helical compression spring

$$F.L. = (i_t - 1) \times Gap + i_t d + y$$

Compressed length:

The axial length of the spring which is subjected to maximum compressive force

$$C.L. = (i_t - 1) \times Gap + S.L$$

Solid Length:

The axial length of the spring which is so compressed that the adjacent coils touch each other

$$S.L. = i_t d$$

Terminology of Springs

Pitch of the coil (p): Axial distance between adjacent coils in uncompressed state of spring

$$p = \frac{Free\ Length}{(i_t - 1)}$$

Stiffness of the spring (k): Force required to produce unit deflection

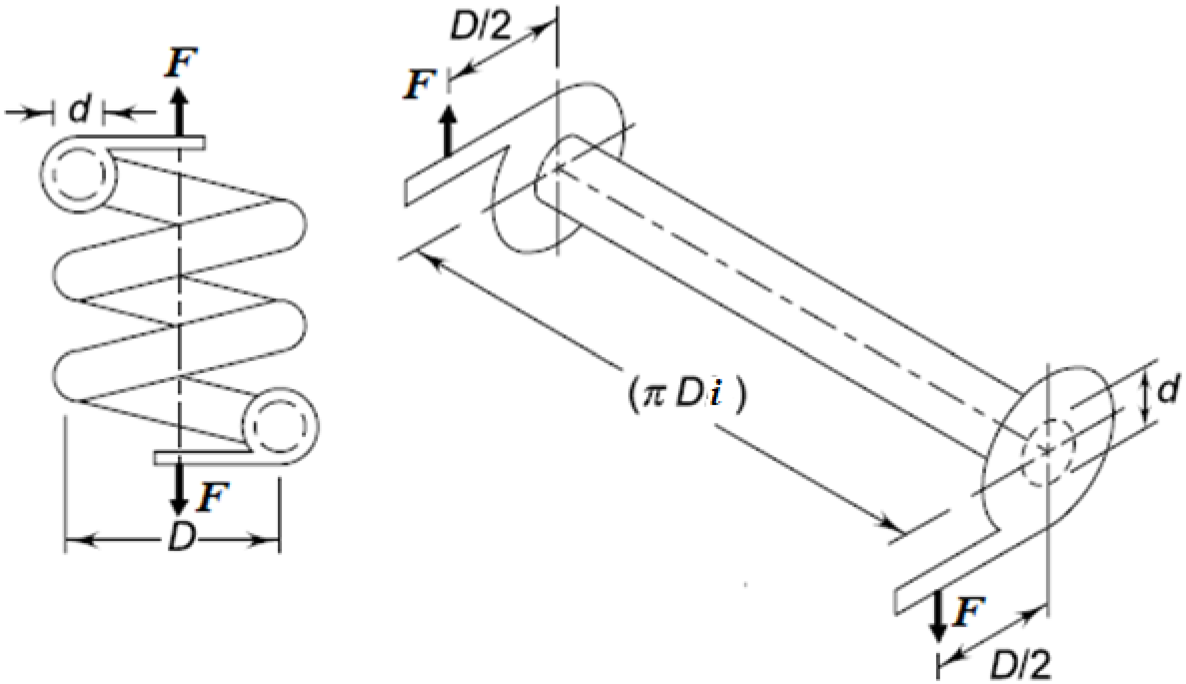
$$k = \frac{F}{y} \qquad \text{Eqn 11.7a}$$

Active (N) and inactive coils: Active coils are the coils in the spring which contribute to spring action, support the external force and deflect under the action of force

End coils which do not contribute to spring action are called inactive coils

$$Inactive\ coils = (i_t - i)$$

Unbent Spring



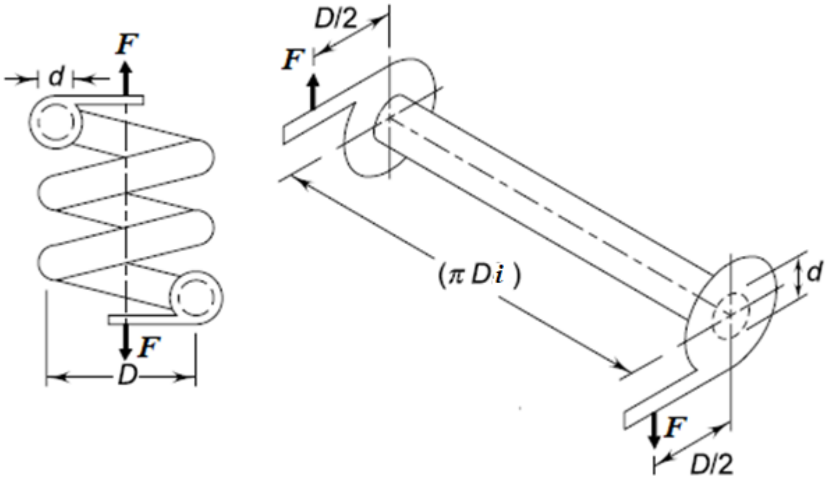
Two basic equations for the design of helical springs

- Load-stress equation
- Load-deflection equation

Dimensions

- The diameter of the bar is equal to the wire diameter of the spring (d)
- The length of the equivalent bar is $(\pi D i)$
- Bar is fitted with bracket of length (D/2)

Unbent Spring



Torsional moment due to force P

$$M_t = \frac{FD}{2}$$

Torsional shear stress due to the force P

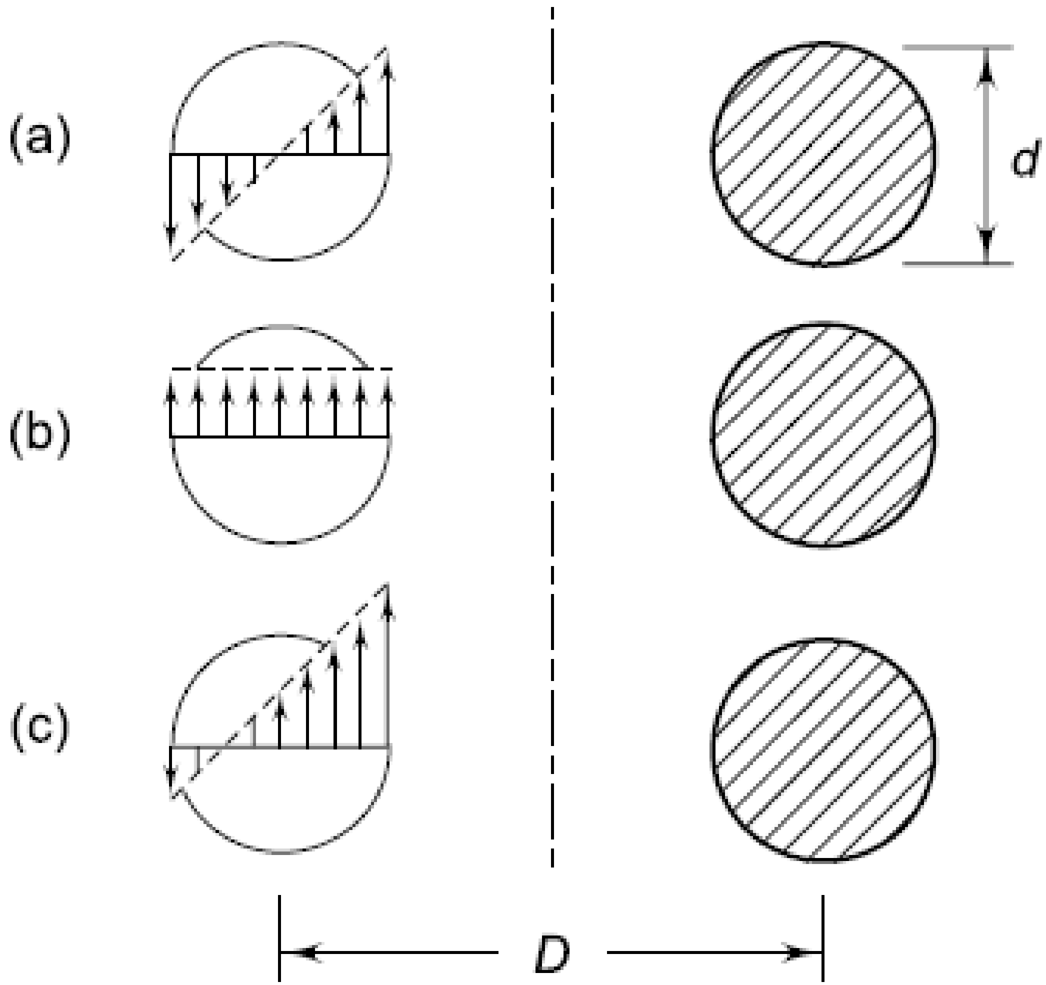
$$\tau_1 = \frac{Tr}{J} = \frac{G\theta}{l} = \frac{8FD}{\pi d^3}$$

Eqn 11.1c

Considering equivalent bar in the form of helical coil, the additional stresses accounted

- Direct or transverse shear stress in spring wire
- Length of inside fibre is less than length of outside fibre which induces stress concentration at the inside of fibre coil

Stresses in spring wire



Pure torsional stress

Direct shear stress

Combined torsional, direct and curvature shear stress

Load-Stress Equation

Modification in the stress equation to accommodate the direct shear and curvature stress effect

$K=K_sK_c$

Direct shear stress, $\tau_2 = \frac{4F}{\pi d^2} = \frac{8FD}{\pi d^3}\left(\frac{0.5d}{D}\right)$

$$\tau \quad = \tau_1 + \tau_2 = \frac{8FD}{\pi d^3}\left(1 + \frac{0.5d}{D}\right) = K_s\frac{8FD}{\pi d^3}$$

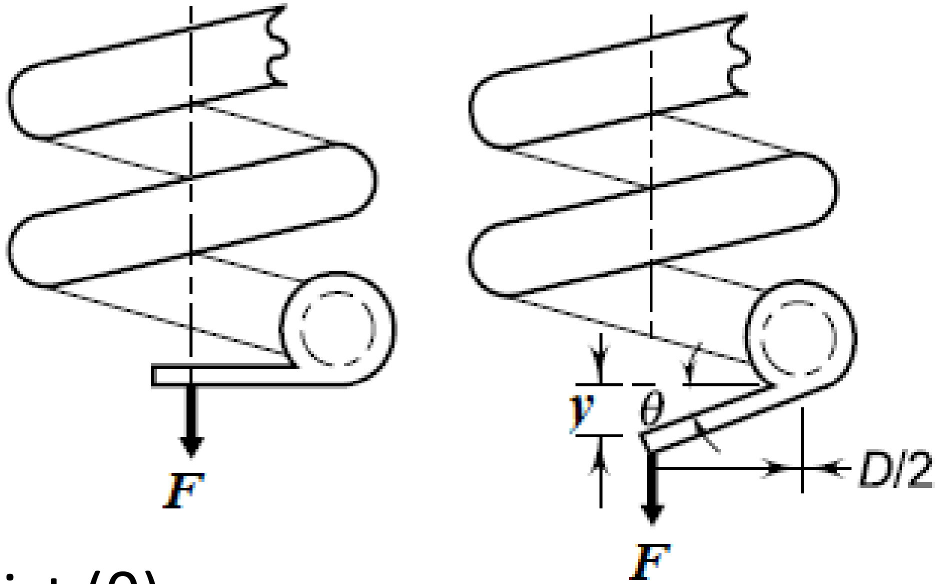
Combined torsional, direct and curvature shear stress by Wahl

$$\tau = K\frac{8FD}{\pi d^3}$$ Eqn 11.1c

Wahl factor $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ Eq 11.2a

LOAD-STRESS EQUATION

Load-Deflection Equation



The angle of twist (θ)

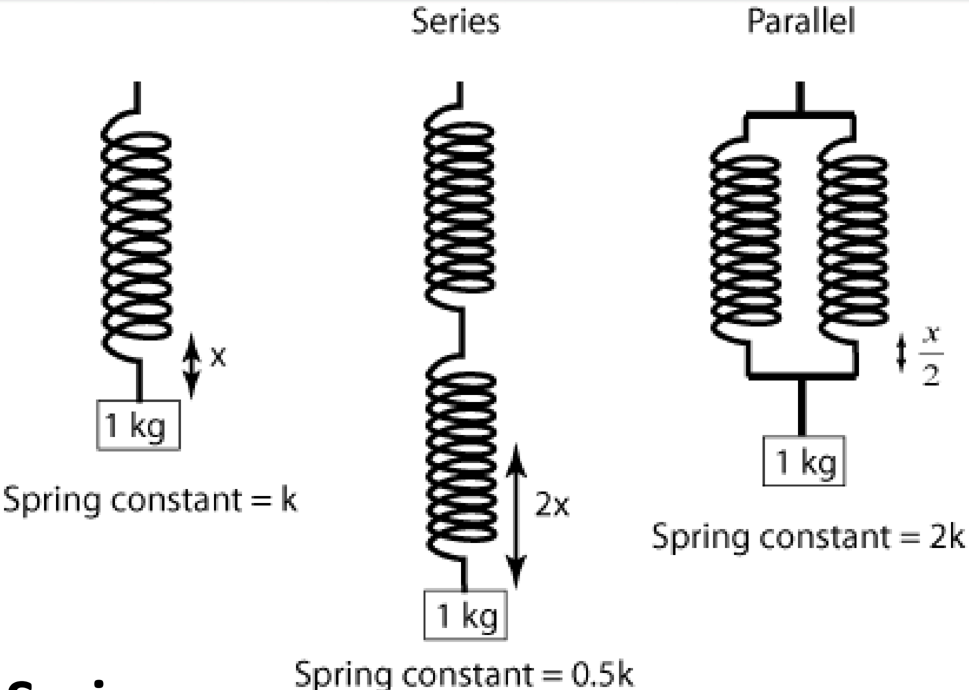
$$\theta = \frac{M_t l}{JG} = \frac{16FD^2 i}{Gd^4} \quad \text{Eqn 11.4}$$

Axial deflection, $y = \dfrac{8FD^3 i}{Gd^4}$ Eqn 11.5a

LOAD-DEFLECTION EQUATION

Stiffness of material, $k = \dfrac{Gd^4}{8D^3 i}$ Eqn 11.7a

Series and Parallel Connections



Parallel:

- Force acting is sum of forces acting on individual spring ($F = F_1 + F_2$)
- Total deflection will be same (δ)

$$y_1 k_1 = F \qquad y_2 k_2 = F$$

$$\boxed{k = k_1 + k_2 + ..}$$

Series:

- Force acting is same (F)
- Total deflection will be sum of deflections of individual springs $y = y_1 + y_2$

$$y_1 = \frac{F}{k_1} \qquad y_2 = \frac{F}{k_2}$$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + ..}$$

Helical Spring: Design Procedure

- Estimate spring force (F) and required deflection (y), In some cases it will be specified
- Select suitable material, Obtain permissible shear stress, $\tau = 0.5S_{ut}$
- Assume spring index value C, Preferred (8-10)
- Calculate Wahl factor, K (Eqn. 11.2a)
- Determine wire diameter from load-stress equation, d (Eqn 11.1d)
- Determine coil diameter, D, (eqn 11.2c)
- Determine number of active coils, i by load deflection equation, i (eqn 11.5a)
- Determine total number of coils, i_t, (table 11.4)
- Determine solid length of the spring, (table 11.4)
- Find actual deflection, y by load-deflection equation and also free length (Assuming suitable gaps)
- Obtain pitch of the coil, p (table 11.4)
- Determine actual spring rate, k (eqn 11.7a)

Problem 5.1

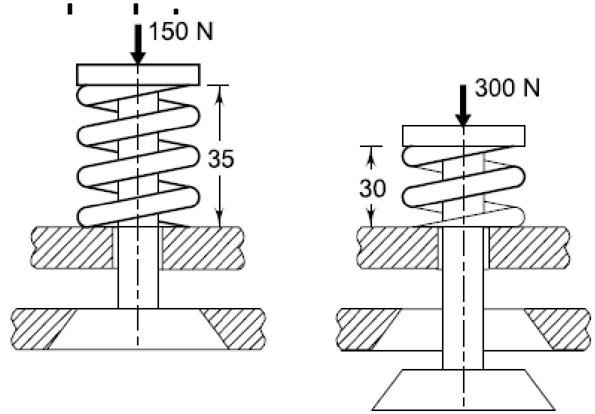
It is required to design a helical compression spring subjected to a maximum force of 1250 N. The deflection of the spring corresponding to the maximum force should be approximately 30 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81 370 N/mm^2, respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate

- Wire diameter
- Mean coil diameter
- Number of active coils
- Total number of coils
- Free length of the spring
- Pitch of the coil

Problem 5.2

It is required to design a helical compression spring for the valve mechanism. The axial force acting on the spring is 300 N when the valve is open and 150 N when the valve is closed. The length of the spring is 30 mm when valve is open and 35 mm when the valve is closed. The spring is made of oil-hardened and tempered valve spring wire and the ultimate tensile strength is 1370 N/mm^2. The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. The modulus of rigidity is 81370N/mm^2. The spring is to be fitted over a valve rod and the minimum inside diameter of the spring should be 20 mm. Design the spring and

- Wire diameter
- Mean coil diameter
- Number of active coils
- Total number of coils
- Free length of the spring
- Pitch of the coil



(a) Closed position (b) Open position

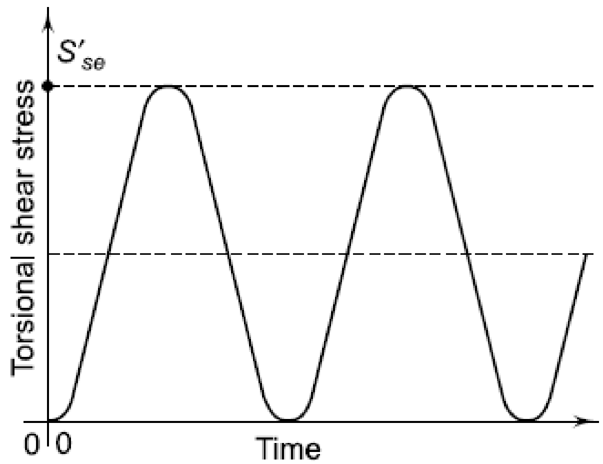
Assume that the clearance between adjacent coils or clash allowance is 15% of the deflection under the maximum load

Design against fluctuating load

Mean force, $F_m = \frac{1}{2}(F_{max} + F_{min})$ Eqn 11.16f

Amplitude force, $F_a = \frac{1}{2}(F_{max} - F_{min})$ Eqn 11.16g

Torsional Mean stress, $\tau_m = K_s\left(\frac{8F_mD}{\pi d^3}\right)$, $K_s = \left(1 + \frac{0.5}{C}\right)$ Torsional stress amplitude, $\tau_a = K\left(\frac{8F_aD}{\pi d^3}\right)$, $K_s = Wahl\ factor$



Rotating beam completely reversed cycle

Springs are subjected to pulsating shear stresses which vary from 0 to S'_{se}

Design against fluctuating load

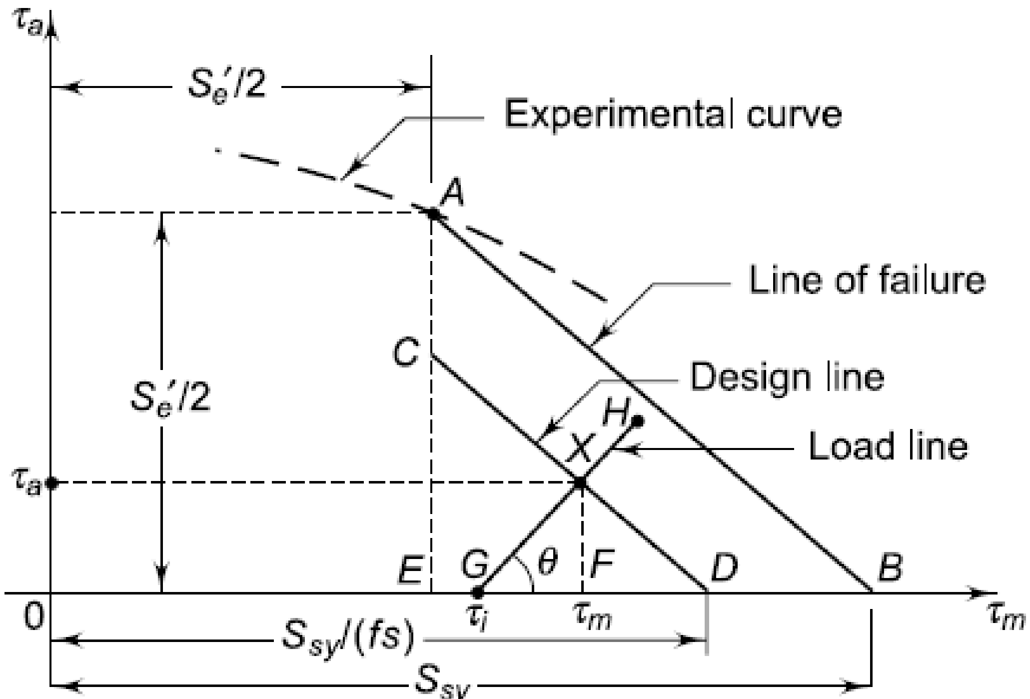
Empirical relationships

- Patented and cold-drawn steel wires

$$S'_{se} = 0.21 S_{ut} \qquad S_{sy} = 0.42 S_{ut}$$

- Oil hardened and tempered steel wires

$$S'_{se} = 0.22 S_{ut} \qquad S_{sy} = 0.45 S_{ut}$$



$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}}$$

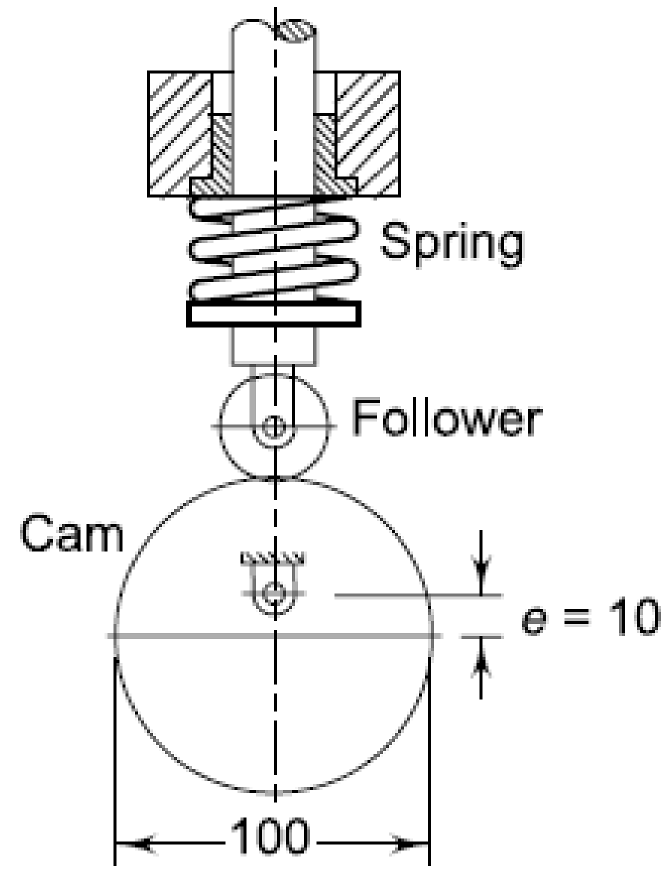
$$\frac{\tau_a}{\frac{S_{sy}}{fs} - \tau_m} = \frac{\frac{1}{2} S'_{se}}{S_{sy} - \frac{1}{2} S'_{se}}$$

Problem 5.3

A helical compression spring of cam mechanism is subjected to an initial preload of 50 N. The maximum operating force during the load cycle is 150 N. The wire diameter is 3mm while the mean coil diameter is 18 mm. The spring is made of oil-hardened and tempered valve spring wire of grade VW (S_{ut}=1430N/mm^2). Determine the factor of safety used in design on the basis of fluctuating stresses.

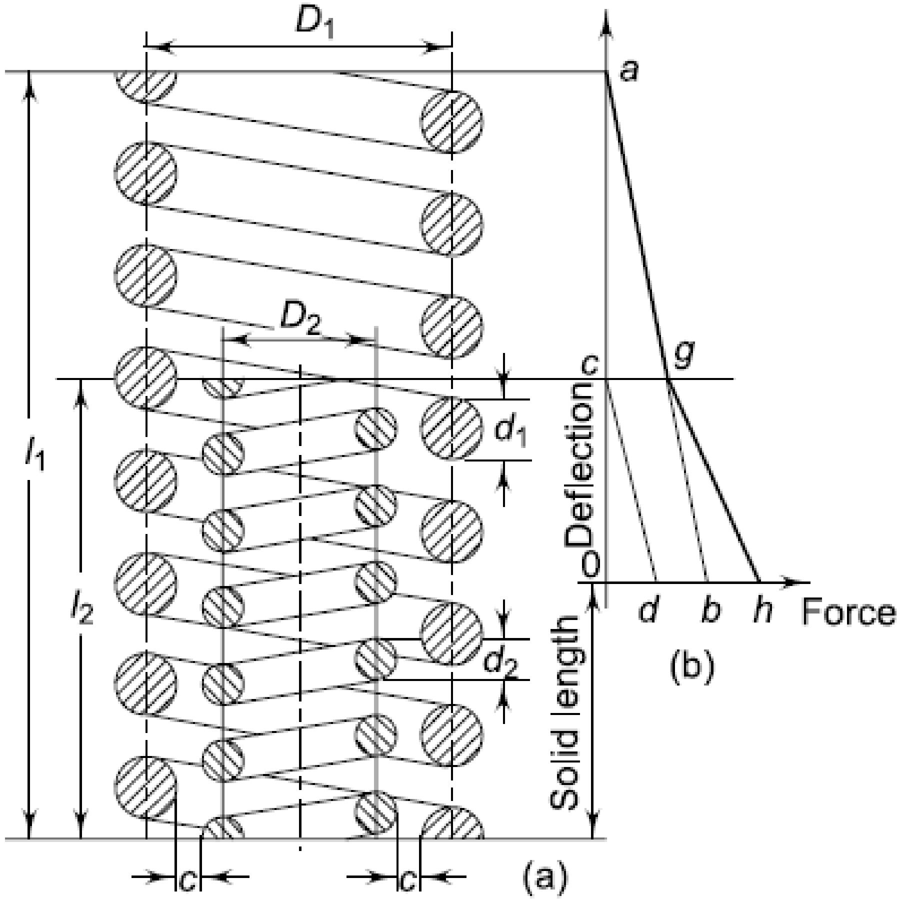
Problem 5.4

An eccentric cam, 100 mm in diameter rotates with an eccentricity of 10 mm as shown in Figure. The roller follower is held against the cam by means of a helical compression spring. The force between the cam and the follower varies from 100 N at the lowest position to 350 N at the highest position of follower. The permissible shear stress in the spring wire is recommended as 30% of ultimate tensile strength. Design the spring from static consideration and determine the factor of safety against fluctuating stresses. Neglect the effect of inertia forces. Assume S_{ut}=1480N/mm^2



Concentric Springs

- Two helical compression springs, one inside the other
- The two consecutive springs will have opposite helix to avoid locking of coils



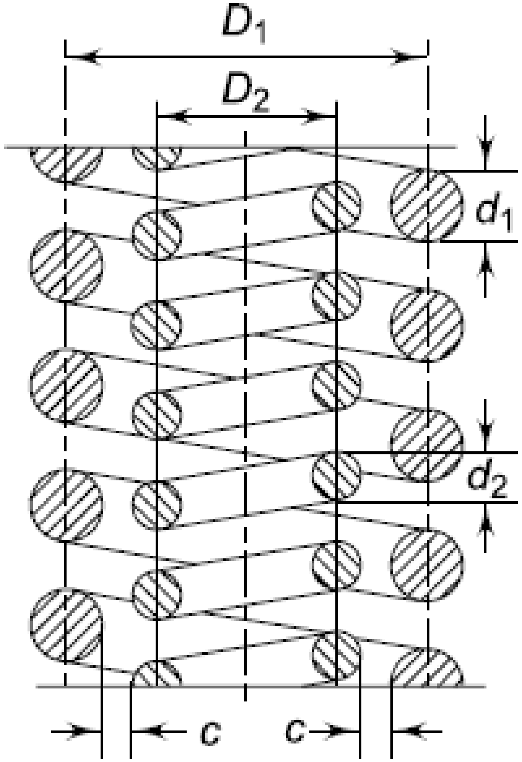
Advantages

- Load carrying capacity is increased
- Fail safe system
- Surge are eliminated

Concentric Springs

Assumptions in Concentric spring

- Springs are made of same material
- Maximum torsional shear stresses induced in outer and inner springs are equal
- Same free length
- Both springs are deflected by same amount



$$\tau_1 = \tau_2 \qquad K_1\left(\frac{8F_1D_1}{\pi d_1^3}\right) = K_2\left(\frac{8F_2D_2}{\pi d_2^3}\right)$$

Assume, $K_1 = K_2$

$$\left(\frac{F_1D_1}{d_1^3}\right) = \left(\frac{F_2D_2}{d_2^3}\right) \qquad \text{Eqn (a)}$$

$$y_1 = y_2 \qquad \left(\frac{F_1D_1^3 i_1}{d_1^4}\right) = \left(\frac{F_2D_2^3 i_2}{d_2^4}\right) \qquad \text{Eqn (b)}$$

Concentric Springs

When both springs are completely compressed, solid length will be equal

$$d_1 i_1 = d_2 i_2$$

$$\left(\frac{F_1 D_1^3 i_1 d_1}{d_1^5}\right) = \left(\frac{F_2 D_2^3 i_2 d_2}{d_2^5}\right) \qquad \text{Eqn (c)}$$

$$\left(\frac{F_1 D_1^3}{d_1^5}\right) = \left(\frac{F_2 D_2^3}{d_2^5}\right)$$

Eqn (c)/Eqn (a)

$$\left(\frac{D_1}{d_1}\right) = \left(\frac{D_2}{d_2}\right) \qquad \text{Eqn (d)}$$

Eqn (d)/Eqn (a)

$$\left(\frac{F_1}{d_1^2}\right) = \left(\frac{F_2}{d_2^2}\right)$$

$$\left(\frac{F_1}{F_2}\right) = \left(\frac{a_1}{a_2}\right)$$

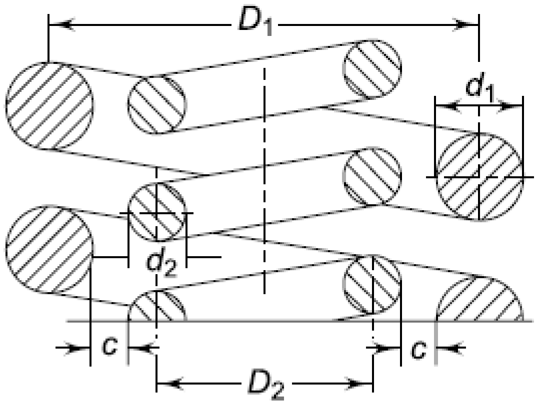
Concentric Springs

Assume, c as the radial clearance between the springs

$$D_1 = D_2 + \left(\frac{d_2}{2} + \frac{d_2}{2}\right) + 2c + \left(\frac{d_1}{2} + \frac{d_1}{2}\right)$$

$$2c = (D_1 - D_2) - (d_1 + d_2)$$

$$c = \frac{(D_1 - D_2)}{2} - \frac{(d_1 + d_2)}{2}$$



Assuming diametric clearance (2c) as difference between wire diameters

$$2c = (d_1 - d_2)$$

$$c = \frac{(d_1 - d_2)}{2}$$

$$\frac{(d_1 - d_2)}{2} = \frac{(D_1 - D_2)}{2} - \frac{(d_1 + d_2)}{2}$$

$$(D_1 - D_2) = (d_1 - d_2) + (d_1 + d_2) = 2d_1$$

$$Take, D_1 = Cd_1 \& D_2 = Cd_2$$

$$\frac{d_1}{d_2} = \frac{C}{C-2}$$

Problem 5.5

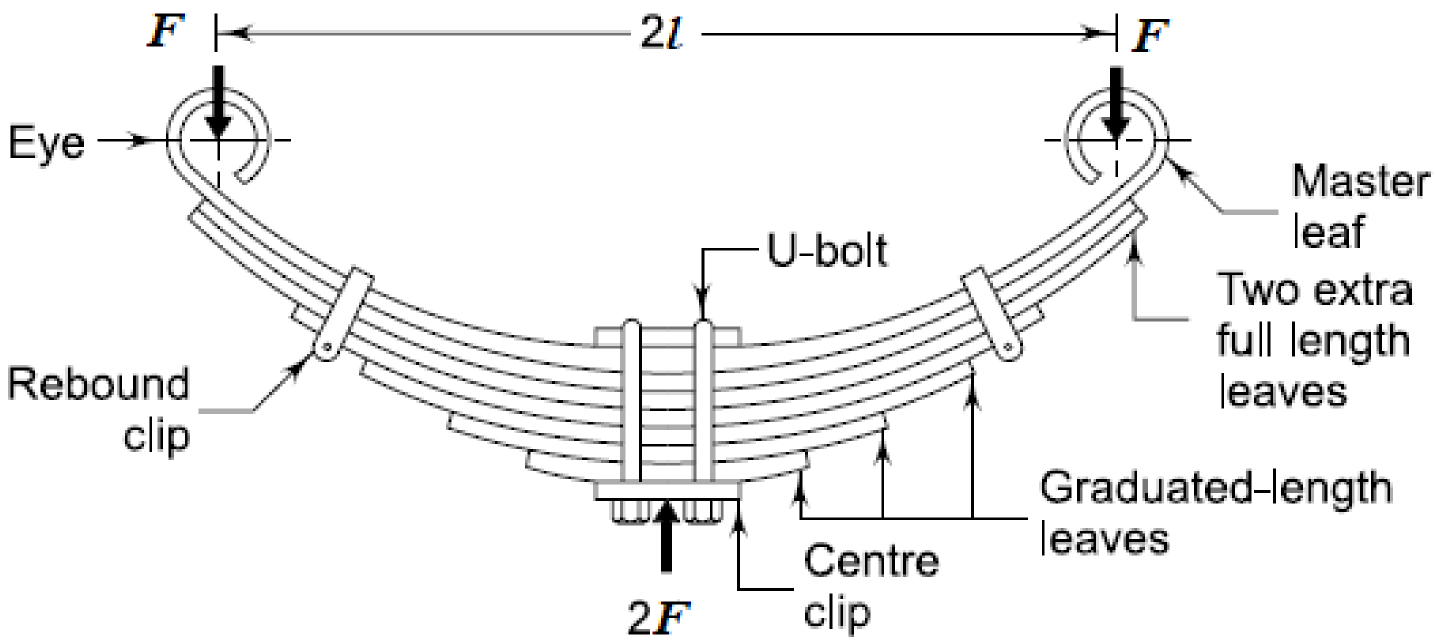
A concentric spring is used as a valve spring in a heavy duty diesel engine. It consists of two helical compression springs having the same free length and same solid length. The composite spring is subjected to a maximum force of 6000N and the corresponding deflection is 50 mm. The maximum torsional shear stress induced in each spring is 800 N/mm^2. The spring index of each spring is 6. Assume same material for two springs and the modulus of rigidity is 81370 N/mm^2. The diametral clearance between the coils is equal to the difference between their wire diameters. Calculate:

- The axial force transmitted by each spring
- Wire and mean coil diameters of each spring
- Number of active coils in each spring

Surge in Spring

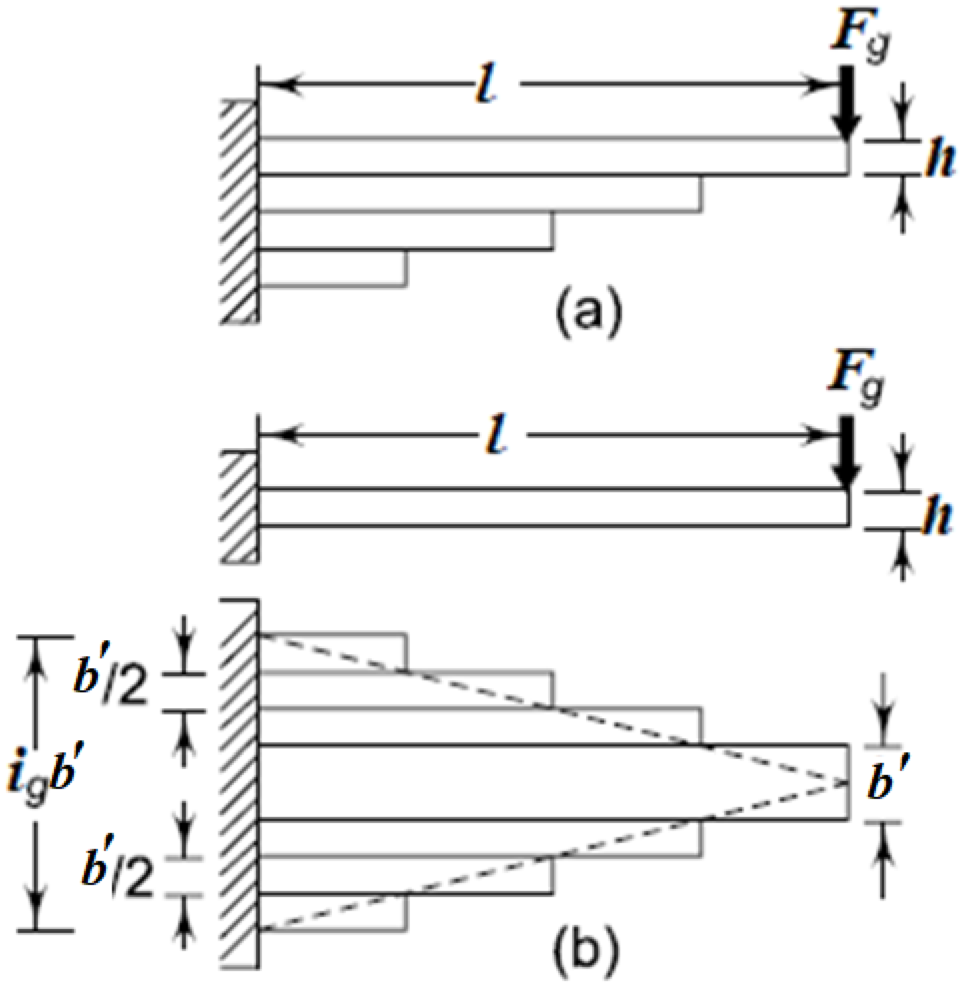
- Natural frequency of vibration of spring coincides with the frequency of external periodic force, resonance occurs
- Spring is subjected to a wave of successive compressions that travels from one end to other and back which is termed as surge
- Load is transmitted by transferring compression to adjacent coils
- If the onward and backward travel time coincides with exiting frequency, resonance occurs
- Natural frequency of helical compression springs (between two parallel plates), $\omega = \frac{1}{2}\sqrt{\frac{k}{m}}$

Leaf Springs



- Consists of flat plates/ leaves
- Longest leaf at top is called master leaf
- Bent at the end to form spring eyes
- U bolt and centre clip to hold the leaf together
- Master leaf, extra full length leaf, graduated leaf

Leaf Springs: Graduated leaves



Load- Stress equation

$$\sigma = \frac{6F_g l}{i_g b h^2}$$

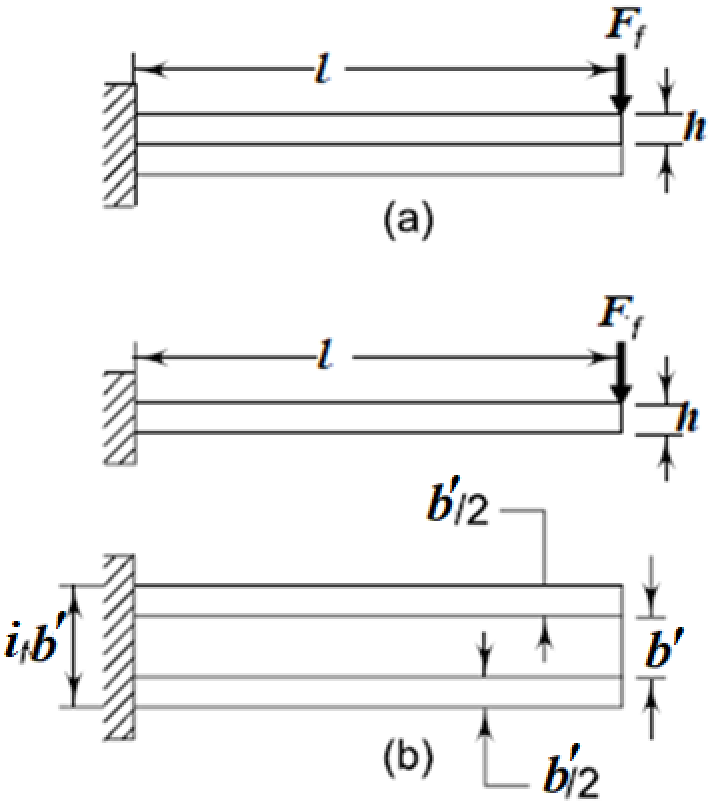
Eqn 11.28b
Table 11.9

Load- Deflection equation

$$y = \frac{6F_g l^3}{E i_g b h^3}$$

Eqn 11.28c
Table 11.9

Leaf Springs: Extra full length leaves



Load- Stress equation

$$\sigma = \frac{6F_f l}{i_f b h^2}$$

Eqn 11.28b
Table 11.9

Load- Deflection equation

$$y = \frac{4F_f l^3}{E i_f b h^3}$$

Eqn 11.28c
Table 11.9

Taking $y_f = y_g \; and \; F_g + F_f = F$ Eqn 11.29d

$$(\sigma)_f = \frac{18Fl}{b'h^2(3i_f + 2i_g)}$$

Eqn 11.30a

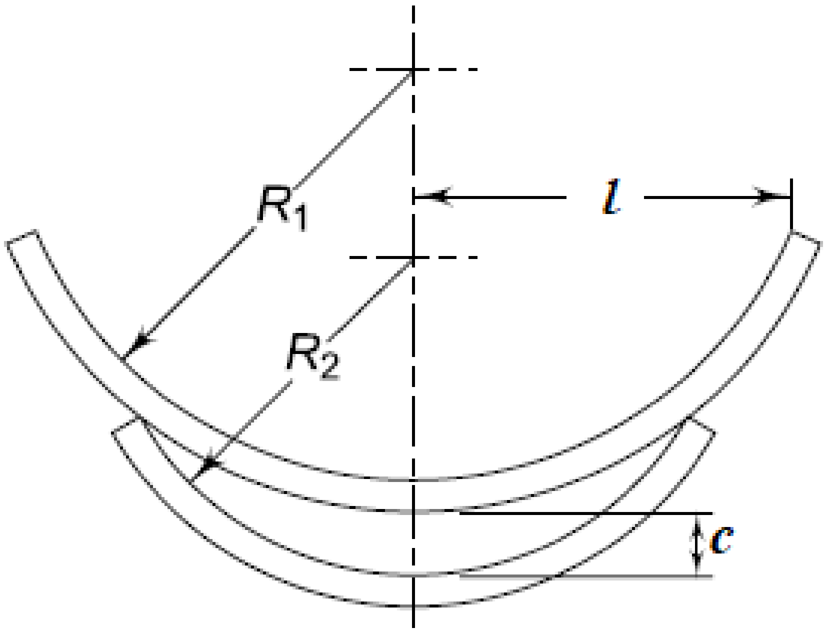
$$(\sigma)_g = \frac{12Fl}{b'h^2(3i_f + 2i_g)}$$

Eqn 11.30b

$$y = \frac{12Fl^3}{b'h^3E(3i_f + 2i_g)}$$

Eqn 11.30c

Nipping of leaf springs



- Pre-stressing by bending leaves to a different radii
- Initial gap between full length leaf and graduated length leaf before the assembly is called nip

$$(\sigma_b)_g = \frac{6F_g l}{i_g b' h^2} \qquad (\sigma_b)_f = \frac{6F_f l}{i_f b' h^2}$$

Taking $c = y_g - y_f, Equalising\ stress\ values$

$$c = \frac{2Fl^3}{Eib'h^3}$$ Eqn 11.32a

$$(\sigma_b) = \frac{6FL}{ib'h^2}$$ Eqn 11.32c

$$F_i = \frac{2i_g i_f F}{i(2i_g + 3i_f)}$$ Eqn 11.32b

Problem 5.7

A semi elliptic leaf spring used for automobile suspension consists of three extra full length leaves and 15 graduated length leaves, including the master leaf. The centre-to-centre distance between two eyes of the spring is 1m. The maximum force that can act on the spring is 75kN. For each leaf, the ratio of width-to-thickness is 9:1. The modulus of elasticity of the leaf material is 207000N/mm^2. The leaves are pre-stressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 N/mm^2.

Determine

- Width and thickness of the leaves
- The initial nip

Problem 5.8

A semi elliptical laminated vehicle spring to carry a load of 6000N is to consist of seven leaves, 65 mm wide. Two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa.

Determine

- The thickness of the leaves
- Deflection of the spring